Math 121 - Test 2 (Practice)   **Name** _____
October 22, 2018

No calculators. Circle or box-in answers.

1. Fill in the blanks:

(a) Let f be a one-to-one function with domain A and range B. Then its inverse function f^{-1} has domain
_____ and range _____ and is defined by
$$f^{-1}(y) = x \iff \underline{\hspace{2cm}}$$
for any y in B. From this follows the cancellation equation
$$f(f^{-1}(x)) = \underline{\hspace{1cm}}$$
and it holds for every x _____.   (5 pts.)

(b) $\log_b b^x = $ _____ for every x in _____.   (2 pts.)

(c) Let a and b be positive numbers. Then $\ln\left(\dfrac{a}{b}\right) = $ _____.   (2 pts.)

(d) A function f is continuous at $x = 5$ if _____.   (2 pts.)

(e) A function f is said to be differentiable at $x = 5$ if _____. (2 pts.)

(f) According to the the Quotient Law for limits, $\lim_{x \to a} \dfrac{f(x)}{g(x)} = $ _____
if _____.   (2 pts.)

(g) According to the change of base formula, $\log_3 7 = $ _____.   (1 pt.)

2. Solve the following equation for x: $e^{7-4x} = 6$.   (2 pts.)

3. Solve the following equation for x: $\ln x + \ln(x-1) = 1$.   (2 pts.)

4. Is there anything wrong with the following equation: $\dfrac{x^2 - 2x - 3}{x + 1} = x - 3$? Explain.

5. Use the Limit Laws to evaluate each of the following limits, if it exists. (<u>Note</u>. Even if an answer is correct, there will be no credit if a law is used incorrectly. Furthermore, points will be deducted for misuse of notation, such as misuse of the $=$ symbol.)   (25 pts.)

$$\lim_{x \to -1} x^4 - 2x^3 - 3$$

$$\lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1}$$

$$\lim_{x \to 3} \sqrt{\frac{25 - x^2}{7 - x}}$$

$$\lim_{x \to 4} \frac{x^2 - 16}{4 - x}$$

$$\lim_{x \to -\pi} \cos x$$

6. (a) Explain what is wrong with the following argument? (5 pts.)

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = \lim_{x \to 0} x^2 \cdot \lim_{x \to 0} \sin\left(\frac{1}{x}\right) = 0.$$

(b) Show $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ correctly and name the limit result(s) that you are using. (5 pts.)

7. Let $h(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 2x + k & \text{if } x > 2. \end{cases}$

(a) Find $\lim_{x \to 2^-} h(x)$. (5 pts.)

(b) Find the value of k so that $\lim_{x \to 2} h(x)$ exists. (5 pts.)

8. Explain why the function
$$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0. \end{cases}$$
is discontinuous at $x = 0$.

9. Find the derivative $f'(a)$ for the function $f(x) = 5x^2 - 3x + 4$ using the <u>definition</u> of the derivative. (No credit will be given for using differentiation rules that you may have learned in other courses.)

10. A particle moves along a horizontal line with its position given by $x = \dfrac{112}{t+2}$ meters where t denotes the time in seconds. Find the velocity and the speed of the particle when $t = 2$ seconds.